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M-THEORY DUALITY AND BPS-EXTENDED SUPERGRAVITY

BERNARD DE WIT
Institute for Theoretical Physics and Spinoza Institute
Utrecht University, Utrecht, The Netherlands

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We discuss toroidal compactifications of maximal supergravity coupled to an extended configuration of BPS states which transform consistently under the U-duality group. Under certain conditions this leads to theories that live in more than eleven spacetime dimensions, with maximal supersymmetry but only partial Lorentz invariance. We demonstrate certain features of this construction for the case of nine-dimensional $N = 2$ supergravity. [ITP-UU-00/28; SPIN-00/26]

1. Introduction

Not too much is known about the underlying degrees of freedom of M-theory. From perturbative string theory one knows about the existence of massive and massless string states. The latter are captured in supergravity where one can explore solitonic solutions which correspond to the branes that arise in (nonperturbative) string theory. In spacetimes with compactified dimensions there exist winding and momentum states. These are BPS states, which can in principle be incorporated into supergravity as matter supermultiplets. As with all BPS states, one has a rather precise knowledge about their mass spectrum and multiplicity, which suffices for many applications. Massive string states decouple in the zero-slope limit; some of them are also BPS. More daring ideas about the underlying degrees of freedom of M-theory are based on supermembrane and matrix theories[1,2,3] (for recent reviews, see refs. 4).

M-theory is subject to a large number of duality symmetries and equivalences. From the string perspective one knows about S- and T-duality which are conjectured[5] to be contained into the so-called U-duality group. This is an arithmetic subgroup of the group of nonlinearly realized symmetries (henceforth denoted by G) of the maximal supergravity theories[6]. Hence the U-dualities coincide with the geometrical symmetries of supergravity. They are conjectured to be exact symmetries of (toroidally compactified) M-theory and therefore act on the BPS states. For a comprehensive review, see ref. 7. The group G is expected to be broken down to an integer-valued subgroup because it must respect the (perturbatively or nonperturbatively generated) charge lattice of the BPS states. It is unavoidable that the inclusion of BPS states leads us back (at least in special limits) to eleven-

dimensional supergravity[8] or extensions thereof, for the simple reason that the BPS states include the Kaluza-Klein (KK) states needed to elevate the theory to eleven dimensions. Of course, a multitude of arguments has already been presented all pointing to the conclusion that eleven-dimensional supergravity has a role to play (in particular, see refs. 5,9,10).

Clearly, an important distinction between toroidally compactified M-theory and toroidally compactified eleven-dimensional supergravity resides in the BPS states. The compactifications of M-theory on a torus T^n are conjectured to be invariant under the duality group $\mathrm{E}_{n(n)}(\mathbf{Z})$, which includes the $\mathrm{SL}(2,\mathbf{Z})$ S-duality group of the IIB superstring as a subgroup. This conjecture includes the BPS states which must transform according to the same duality group. In contrast with this, the toroidal compactifications of eleven-dimensional supergravity generate towers of KK states whose central charges are associated with momenta in the compactified dimensions. These states cannot transform under the duality symmetry, simply because the central charges that they carry are too restricted; central charges associated with the two- and five-brane charges in eleven dimensions vanish for the KK states. Obviously the massless states coincide for the two theories. But in view of the fact that M-theory contains more BPS states than the KK states associated with the momenta on the hyper-torus, one may wonder why the spacetime dimension should remain restricted to eleven. This is one of the questions that we will address here.

In principle, the central charges transform according to the automorphism group $\mathrm{H_R}$ of the supersymmetry algebra that rotates the supercharges and commutes with the D-dimensional Lorentz group, where $D = 11 - n$. In most cases, however, the central charges can also be assigned to the larger U-duality group. However, there are exceptions for spacetime dimensions $D = 11 - n \leq 5$ for the low-dimensional branes (for a discussion, see refs. 7,11).On the other hand we note that the anti-symmetric gauge fields of supergravity are assigned to representations of G and not of $\mathrm{H_R}$ (the only exceptions exist in $D = 4, 8$ where one has to rely on the Hodge duality rotations of the field strengths in order to realize the full duality group).

When the central charges, and thus the corresponding BPS states that carry those charges, constitute representations of the U-duality group, it may be possible to incorporate all these BPS states into a local supergravity field theory in a way that is U-duality invariant. We call such theories BPS-extended supergravities. The construction of these theories is motivatied in part by some old intriguing result that the toroidal compactifications of eleven-dimensional supergravity, without truncation to the massless modes, exhibit traces of the hidden symmetries[12,13], while one of the reasons for the lack of invariance of the untruncated theory is precisely the incompleteness of the massive KK states with respect to the hidden symmetries. Interestingly enough, by insisting on U-duality for the BPS states one usually goes beyond eleven-dimensional supergravity. To study the properties of these new theories we discuss the simplest version corresponding to a toroidal compactification to nine spacetime dimensions, where we couple a variety of BPS supermultiplets to the unique $N = 2$ supergravity theory, which is thus based on 32 supercharges. The

supergravity theory has been studied previously[14], but the main emphasis here is on the BPS states. We also make contact with earlier work[15] where it was argued that the BPS multiplets have a natural interpretation in terms of the momentum and wrapping states on the M-theory torus. The IIA/B T-duality and the IIA/M S-duality combines as a duality between IIB theory on $R^9 \times S^1$ and M-theory on $R^9 \times T^2$. As we were alluding to above, this theory can be interpreted as a theory living in twelve dimensions (although there is no twelve-dimensional Lorentz invariance). In special decompactification limits it coincides with eleven-dimensional or IIA/B supergravity[16]. In the next sections we discuss various aspects of this nine-dimensional BPS-extended theory.

2. Maximal Supersymmetry and BPS States in Nine Dimensions

Let us start by considering the BPS multiplets that are relevant in nine spacetime dimensions from the perspective of supergravity, string theory and (super)membranes. It is well known that the massive supermultiplets of IIA and IIB string theory coincide, whereas the massless states comprise inequivalent supermultiplets, for the simple reason that they transform according to different representations of the SO(8) helicity group. When compactifying the theory on a circle, massless IIA and IIB states in nine spacetime dimensions transform according to identical SO(7) representations of the helicity group and constitute equivalent supermultiplets. The corresponding interacting field theory is the unique $N = 2$ supergravity theory in nine spacetime dimensions. However, the supermultiplets of the BPS states, which carry momentum along the circle, remain inequivalent, as they remain assigned to the inequivalent representations of the group SO(8) which is now associated with the restframe (spin) rotations of the massive states. The momentum states of the IIA and the IIB theories will be denoted henceforth as KKA and KKB states, respectively. The fact that they constitute inequivalent supermultiplets, has implications for the winding states in order that T-duality will remain valid. This is discussed below.

In ref. 16 this question was investigated in detail and considered in the context of $N = 2$ supersymmetry in nine spacetime dimensions with Lorentz-invariant central charges. These central charges are encoded in a two-by-two real symmetric matrix Z^{ij}, which can be decomposed as

$$Z^{ij} = b\,\delta^{ij} + a\,(\cos\theta\,\sigma_3 + \sin\theta\,\sigma_1)^{ij}\,. \qquad (1)$$

Here $\sigma_{1,3}$ are the real symmetric Pauli matrices. We note that the central charge associated with the parameter a transforms as a doublet under the SO(2) group that rotates the two supercharge spinors, while the central charge proportional to the parameter b is SO(2) invariant. Subsequently one shows that BPS states that carry these charges must satisfy the mass formula,

$$M_{\mathrm{BPS}} = |a| + |b|\,. \qquad (2)$$

Here one can distinguish three types of BPS supermultiplets. One type has central

charges $b=0$ and $a\neq 0$. These are 1/2-BPS muliplets, because they are annihilated by half of the supercharges. The KKA supermultiplets that comprise KK states of IIA supergravity are of this type. Another type of 1/2-BPS mulitplets has central charges $a=0$ and $b\neq 0$. The KKB supermultiplets that comprise the KK states of IIB supergravity are of this type. Finally there are 1/4-BPS multiplets (annihilated by one fourth of the supercharges) characterized by the fact that neither a nor b vanishes.

For type-II string theory one obtains these central charges in terms of the left- and right-moving momenta, $p_{\rm L}$, $p_{\rm R}$, that characterize winding and momentum along S^1. However, the result takes a different form for the IIA and the IIB theory,

$$Z^{ij} = \begin{cases} \frac{1}{2}(p_{\rm L}+p_{\rm R})\delta^{ij} + \frac{1}{2}(p_{\rm L}-p_{\rm R})\sigma_3^{ij}\,, & \text{(for IIB)} \\ \frac{1}{2}(p_{\rm L}-p_{\rm R})\delta^{ij} + \frac{1}{2}(p_{\rm L}+p_{\rm R})\sigma_3^{ij}\,. & \text{(for IIA)} \end{cases} \tag{3}$$

The corresponding BPS mass formula is equal to

$$M_{\rm BPS} = \tfrac{1}{2}|p_{\rm L}+p_{\rm R}| + \tfrac{1}{2}|p_{\rm L}-p_{\rm R}|\,. \tag{4}$$

For $p_{\rm L}=p_{\rm R}$ we confirm the original identification of the momentum states, namely that IIA momentum states constitute KKA supermultiplets, while IIB momentum states constitute KKB supermultiplets. For the winding states, where $p_{\rm L}=-p_{\rm R}$, one obtains the opposite result: IIA winding states constitute KKB supermultiplets, while IIB winding states constitute KKA supermultiplets. The 1/4-BPS multiplets arise for string states that have either right- or left-moving oscillator states, so that either $M_{\rm BPS}=|p_{\rm L}|$ or $|p_{\rm R}|$ with $p_{\rm L}^2\neq p_{\rm R}^2$. All of this is entirely consistent with T-duality[17,18], according to which there exists a IIA and a IIB perspective, with decompactification radii are that inversely proportional and with an interchange of winding and momentum states.

It is also possible to view the central charges from the perspective of the eleven-dimensional (super)membrane. Assuming that the two-brane charge takes values in the compact coordinates labeled by 9 and 10, which can be generated by wrapping the membrane over the corresponding T^2, one readily finds the expression,

$$Z^{ij} = Z_{9\,10}\,\delta^{ij} - (P_9\,\sigma_3^{ij} - P_{10}\,\sigma_1^{ij})\,. \tag{5}$$

When compactifying on a torus with modular parameter τ and area A, the BPS mass formula takes the form

$$\begin{aligned} M_{\rm BPS} &= \sqrt{P_9^2+P_{10}^2} + |Z_{9\,10}| \\ &= \frac{1}{\sqrt{A\,\tau_2}}|q_1+\tau\,q_2| + T_{\rm m}A\,|p|\,. \end{aligned} \tag{6}$$

Here $q_{1,2}$ denote the momentum numbers on the torus and p is the number of times the membrane is wrapped over the torus; $T_{\rm m}$ denotes the supermembrane tension. Clearly the KKA states correspond to the momentum modes on T^2 while the KKB

states are associated with the wrapped membranes on the torus. Therefore there is a rather natural way to describe the IIA and IIB momentum and winding states starting from a (super)membrane in eleven spacetime dimensions. This point was first emphasized in ref. 15.

3. BPS-extended Supergravity

It is possible to consider $N = 2$ supergravity in nine spacetime dimensions and couple it to the simplest BPS supermultiplets corresponding to the KKA and KKB states. As shown in the previous section there are three central charges and nine-dimensional supergravity possesses precisely three gauge fields that couple to these charges. From the perspective of eleven-dimensional supergravity compactified on T^2, the KK states transform as KKA multiplets. Their charges transform obviously with respect to an SO(2) associated with rotations of the coordinates labeled by 9 and 10. Hence we have a double tower of these charges with corresponding KKA supermultiplets. On the other hand, from the perspective of IIB compactified on S^1, the KK states constitute KKB multiplets and their charge is SO(2) invariant. Here we have a single tower of KKB supermultiplets. However, from the perspective of nine-dimensional supergravity one is led to couple both towers of KKA and KKB supermultiplets simultaneously. In that case one obtains some dichotomic theory[16], which we refer to as BPS-extended supergravity. In the case at hand this new theory describes the ten-dimensional IIA and IIB theories in certain decompactification limits, as well as eleven-dimensional supergravity. But the theory is in some sense truly twelve-dimensional with three compact coordinates, although there is no twelve-dimensional Lorentz invariance, not even in a uniform decompactification limit, as the fields never depend on all the twelve coordinates! Whether this kind of BPS-extended supergravity offers a viable scheme in a more general context than the one we discuss here, is not known. In fact, not much work has been done on incorporating the coupling of BPS multiplets into a supersymmetric field theory. Nevertheless, in the case at hand we know a lot about these couplings from our knowledge of the T^2 compactification of eleven-dimensional supergravity and the S^1 compactification of IIB supergravity.

The fields of nine-dimensional $N = 2$ supergravity are listed in table 1, where we also indicate their relation with the fields of eleven-dimensional and ten-dimensional IIA/B supergravity upon dimensional reduction. It is not necessary to work out all the nonlinear field redefinitions here, as the corresponding fields can be uniquely identified by their scaling weights under SO(1,1), a symmetry of the massless theory that emerges upon dimensional reduction and is associated with scalings of the internal vielbeine. The scalar field σ is related to G_{99}, the IIB metric component in the compactified dimension, by $G_{99} = \exp(\sigma)$; likewise it is related to the determinant of the eleven-dimensional metric in the compactified dimensions, which is equal to $\exp(-\frac{4}{3}\sigma)$. The precise relationship follows from comparing the SO(1,1) weights through the dimensional reduction of IIB and eleven-dimensional supergravity. In nine dimensions supergravity has two more scalars, which are described

Table 1. The bosonic fields of the eleven dimensional, type-IIA, nine-dimensional $N = 2$ and type-IIB supergravity theories. The eleven-dimensional and ten-dimensional indices, respectively, are split as $\hat{M} = (\mu, 9, 10)$ and $M = (\mu, 9)$, where $\mu = 0, 1, \ldots 8$. The last column lists the SO(1,1) scaling weights of the fields.

$D = 11$	IIA	$D = 9$	IIB	SO(1,1)
$\hat{G}_{\mu\nu}$	$G_{\mu\nu}$	$g_{\mu\nu}$	$G_{\mu\nu}$	0
$\hat{A}_{\mu\,9\,10}$	$C_{\mu\,9}$	B_μ	$G_{\mu\,9}$	-4
$\hat{G}_{\mu\,9},\ \hat{G}_{\mu\,10}$	$G_{\mu\,9}\ ,\ C_\mu$	A^α_μ	$A^\alpha_{\mu\,9}$	3
$\hat{A}_{\mu\nu\,9},\ \hat{A}_{\mu\nu\,10}$	$C_{\mu\nu\,9}, C_{\mu\nu}$	$A^\alpha_{\mu\nu}$	$A^\alpha_{\mu\nu}$	-1
$\hat{A}_{\mu\nu\rho}$	$C_{\mu\nu\rho}$	$A_{\mu\nu\rho}$	$A_{\mu\nu\rho\sigma}$	2
$\hat{G}_{9\,10},\ \hat{G}_{9\,9},\ \hat{G}_{10\,10}$	$\phi,\ G_{9\,9},\ C_9$	$\left\{ \begin{array}{l} \phi^\alpha \\ \exp(\sigma) \end{array} \right.$	$\begin{array}{l} \phi^\alpha \\ G_{9\,9} \end{array}$	$\begin{array}{r} 0 \\ 7 \end{array}$

by a nonlinear sigma model based on $\mathrm{SL}(2,\mathbf{R})/\mathrm{SO}(2)$. The coset is described by the complex doublet of fields ϕ^α, which satisfy a constraint $\phi^\alpha \phi_\alpha = 1$ and are subject to a local SO(2) invariance, so that they describe precisely two scalar degrees of freedom ($\alpha = 1, 2$). We expect that the local SO(2) invariance can be incorporated in the full BPS-extended supergravity theory and can be exploited in the construction of the couplings of the various BPS supermultiplets to supergravity.

We already mentioned the three abelian vector gauge fields which couple to the central charges. There are two vector fields A^α_μ, which are the KK photons from the T^2 reduction of eleven-dimensional supergravity and which couple therefore to the KKA states. From the IIA perspective these correspond to the KK states on S^1 and the D0 states. From the IIB side they originate from the tensor fields, which confirms that they couple to the IIB (elementary and D1) winding states. These two fields transform under SL(2), which can be understood from the perspective of the modular transformation on T^2 as well as from the S-duality transformations that rotate the elementary strings with the D1 strings. The third gauge field, denoted by B_μ, is a singlet under SL(2) and is the KK photon on the IIB side, so that it couples to the KKB states. On the IIA side it originates from the IIA tensor field, which is consistent with the fact that the IIA winding states constitute KKB supermultiplets.

From the perspective of the supermembrane, the KKA states are the momentum states on T^2, while the KKB states correspond to the membranes wrapped around the torus. While it is gratifying to see how all these correspondences work out, we stress that from the perspective of nine-dimensional $N = 2$ supergravity, coupled to the smallest 1/2-BPS states, the results follow entirely from supersymmetry.

The resulting BPS-extended theory incorporates eleven-dimensional supergravity and the two type-II supergravities in special decompactification limits. But, as

we stressed above, we are dealing with a twelve-dimensional theory here, of which three coordinates are compact, except that no field can depend on all of the three compact coordinates. The theory has obviously two mass scales associated with the KKA and KKB states. We return to them in a moment. Both S- and T-duality are manifest, although the latter has become trivial as the theory is not based on a specific IIA or IIB perspective. One simply has the freedom to view the theory from a IIA or a IIB perspective and interpret it accordingly.

We should discuss the fate of the group $\mathrm{G} = \mathrm{SO}(1,1) \times \mathrm{SL}(2,\mathbf{R})$ of pure supergravity after coupling the theory to the BPS multiplets. The central charges of the BPS states form a discrete lattice, which is affected by this group. Hence, after coupling to the BPS states, we only have a discrete subgroup that leaves the charge lattice invariant. This is the group $\mathrm{SL}(2,\mathbf{Z})$.

The KKA and KKB states and their interactions with the massless theory can be understood from the perspective of compactified eleven-dimensional and IIB supergravity. In this way we are able to deduce the following BPS mass formula,

$$M_{\mathrm{BPS}}(q_1,q_2,p) = m_{\mathrm{KKA}}\, \mathrm{e}^{3\sigma/7}\, |q_\alpha \phi^\alpha| + m_{\mathrm{KKB}}\, \mathrm{e}^{-4\sigma/7}\, |p| \,, \tag{7}$$

where q_α and p refer to the integer-valued KKA and KKB charges, respectively, and m_{KKA} and m_{KKB} are two independent mass scales. This formula can be compared to the membrane BPS formula (6) in the eleven-dimensional frame. One then finds that

$$m_{\mathrm{KKA}}^2\, m_{\mathrm{KKB}} \propto T_{\mathrm{m}} \,, \tag{8}$$

with a numerical proportionality constant.

4. Supertraces and R^4-terms

In this BPS-extended supergravity theory one can integrate out the BPS supermultiplets and study their contribution to the effective action. In this way one makes contact with the R^4-terms that were considered by refs. 19-21.This amounts to a one-loop calculation in a nine-dimensional field theory. Since we evaluate the coefficient of the R^4 term at zero momentum, the relevant amplitude has the structure of a box diagram in massive φ^3 theory in nine spacetime dimensions (relying on the usual recombination with the non-box graphs that often appears in gauge theories). The contributions from the KKA and KKB states are equal to[22]

$$A_4^{\mathrm{KKA+KKB}} = \frac{1}{(2\pi)^9} \int \mathrm{d}^9 k \sum_{q_1,q_2,p} \frac{1}{[k^2 + M^2(q_1,q_2,p)]^4} \,. \tag{9}$$

The momentum integral is linearly divergent and it is multiplied by a so-called helicity supertrace B_n, with $n = 8$, which correctly counts all the virtual states belonging to the KKA and KKB supermultiplets. These helicity supertraces can be calculated straightforwardly and depend on the supermultiplet of virtual states in the loop. One finds that 1/2-BPS multiplets have a zero supertrace when $n < 8$, the 1/4-BPS multiplets have a zero supertrace when $n < 12$, and the generic

supermultiplets have a zero supertrace for $n < 16$. Therefore only the KKA and KKB supermultiplets are expected to contribute to the R^4-terms (at one loop).

After a Poisson resummation this result can be written as follows (we use the BPS mass formula (6)),

$$\begin{aligned} A_4^{\mathrm{KKA+KKB}} &= \frac{2}{3}\,\frac{A^{-1/2}}{(4\pi)^6} \sum_{q_1',q_2'} \frac{\tau_2^{3/2}}{|q_1' + \tau q_2'|^3} + \frac{4}{3}\,\frac{T_{\mathrm{m}}\, A}{(4\pi)^6} \sum_{p'} \frac{1}{p'^{\,2}} \\ &= \frac{2}{3}\,\frac{1}{(4\pi)^6} \left[A^{-1/2}\, f(\tau,\bar\tau) + \tfrac{2}{3}\pi^2\, T_{\mathrm{m}}\, A\right]. \end{aligned} \tag{10}$$

In the last line we dropped the terms in the sum for $q_1' = q_2' = 0$ and $p' = 0$. These terms represent the ultraviolet divergences. The modular function $f(\tau,\bar\tau)$ is defined by[19]

$$f(\tau,\bar\tau) = \sum_{(q_1',q_2')\neq(0,0)} \frac{\tau_2^{3/2}}{|q_1' + \tau q_2'|^3}\,. \tag{11}$$

This result is invariant (as it should) under the IIB S-duality symmetry $\tau \to (a\tau + b)/(c\tau + d)$ with a, b, c and d integers satisfying $ac - bd = 1$. The contribution from the KKB states is such that the result is compatible with T-duality of type-II string theory.

It is a gratifying result that including the KKA and the KKB states in the context of a uniform regularization scheme leads directly to a result that is consistent with T-duality. Nevertheless, there remains a number of critical questions. The degree of divergence of the initial box graphs is not reduced by supersymmetry, because the relevant helicity supertraces for the KKA and the KKB multiplets, which represent the sum over the virtual states, yield identical multiplicative factors. The subtraction dependence also reflects itself in the fact that the KKA contributions disappear in the decompactification limit to eleven dimensions, $A \to \infty$, which is counterintuitive. Hence the result remains in principle subtraction dependent. In fact the uniform regularization implies that the sum over the KKA and the KKB states leads to double counting of the massless states.

5. Concluding remarks

We have considered extensions of supergravity where one couples to a supersymmetric set of BPS states that is complete with respect to the U-duality group. We used maximal nine-dimensional supergravity as an example, coupled to both KKA and KKB supermultiplets. From the point of view of U-duality, this example is not quite satisfactory in demonstrating all relevant features, as the KKA and the KKB states transform separately under U-duality. In lower-dimensional spacetimes the situation is different and the corresponding BPS states transform irreducibly under U-duality. Nevertheless, in this way one obtains a dichotomic theory that can be regarded as a twelve-dimensional field theory, which, upon suitable decompactification limits, leads to eleven-dimensional supergravity or ten-dimensional IIA/B supergravity.

There are interesting questions regarding the field-theoretic coupling of the fields associated with BPS states. In the case at hand these can in principle be answered, because the couplings can be deduced from the coupling of the massive KK fields in the compactifications of eleven-dimensional and IIB supergravity. One such questions concerns the role of the local symmetry H = SO(2) that one uses in the description of the SL(2)/SO(2) coset space for the nonlinear sigma model. This symmetry has a (composite) gauge field, which does not describe additional degrees of freedom.

What is difficult to answer at present is the question whether such BPS-extended theories can be fully consistent as local field theories. It is known that the coupling of fields of higher spins can lead to minimal-coupling inconsistencies. As long as we consider terms that are at most quadratic in the massive fields, there seems no immediate discrepancy. However, once one includes the higher-order couplings between the BPS states, this is not so clear. Yet, it would be interesting to calculate these couplings as in this way one can make contact with the algebra of BPS states discussed in ref. 23.

As we mentioned before, the extension with BPS states is interesting from the perspective of alternative formulations of eleven-dimensional supergravity which have been written down long ago[12,13]. There it was found that the eleven-dimensional theory does exhibit nontrivial traces of the hidden symmetry groups $E_{7(7)}$ and $E_{8(8)}$, although the invariance under these groups is only realized upon truncating to the massless states. However, to some extent the lack of invariance can be traced back to the incompleteness of the BPS states with respect to U-duality (for a recent discussion, see ref. 11). Therefore it seems reasonable to expect that there do exist interacting field theories of supergravity coupled to BPS states which are consistent and respect U-duality and therefore are suitable starting points for learning more about M-theory. On the other hand, we do expect certain obstacles to this program for the case of toroidal compactifications to low spacetime dimensions. First of all, we already know of some inconsistencies in the U-duality assignments of the central charges, and secondly, the BPS charges will be mutually nonlocal. For instance, the pointlike charges in $D = 4$ spacetime dimensions correspond to electric and magnetic charges, which can not be incorporated in the framework of a local field theory.

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